graphing palindromes

Using graphing to reveal the hidden transformations in palindrome (and other types of) licence plates

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As many students can attest, cars are interesting. They come in various colours, they have interesting accessories, and they move us to where we want to go. Many years ago, the students of Pythagoras thought that numbers were interesting and had a saying that "All is number." As children are taught to embrace mathematics as a dynamic and useful subject, we can show them an interesting context of mathematics where cars and numbers intersect in licence plates.

Hildebrandt, Biglan, and Budd (2013) showed how licence plates are a useful context in which to work with basic number operations, fact family patterns, identifying mnemonic devices to remember licence plates, and creating your own plates. In this article, you will see a range of activities in which to further engage your students using an early form of graphing. While the *Australian Curriculum: Mathematics* (2014) highlights understanding, fluency, problem-solving, and reasoning, the National Research Council (2001) describes five strands of mathematical proficiency, with the additional one being productive disposition. The activities within this article present a way to encourage students to see themselves as creators and interpreters of mathematical concepts, thus developing a strong productive disposition toward mathematics while still addressing required content.

The content covered by these activities fits well with 4th and 5th grade, and with some extensions, up to 8th grade. Table 1 (see next page) shows some of the content standards addressed by the activities presented in this article. In particular, the Year 4 level proficiency strands of understanding and fluency can be emphasised through the symmetrical shapes that are discovered within licence plates, as well as creating shapes and transformations in the collected and recorded data.

In the following sections, we consider some early graphing concepts that easily allow for a new form of graphing licence plates that will lead into some interesting mathematics. Further, in this article, you will see how easily licence plates can be graphed to reveal hidden geometrical transformations. But first, we should discuss palindromes and one-to-one correspondence before starting this new type of graphing.

Palindromes

A palindrome is defined as a word, phrase, or sequence that reads the same backward as forward. In many states and provinces within Australia, the U.S., and Canada, licence plates are represented with a sequence of numbers and letters. If we think of a licence plate as having two discrete sets of characters, one for the set of numbers and the other for the set of letters, we might find licence plates that would fit the definition of two palindromes (see Figure 1). For the transformation activities presented later, we will only be looking at licence plates with the three letters and three numbers format. But before we look in detail at any palindromic plates, let me introduce you to a new form of graphing that many children have found to be quite exciting, and that first requires an understanding of one-to-one correspondence.

Table 1: *Australian Curriculum: Mathematics* (2014) Content descriptions.

Year 4 Content, location and transformation
• Create symmetrical patterns, pictures and shapes with and without digital technologies (ACMMG091).
Year 4 Achievement standard
• Students create symmetrical shapes and patterns. • Students list the probabilities of everyday events. • They construct data displays from given or collected data.
Year 5 Content, location and transformation
• Describe translations, reflections and rotations of two-dimensional shapes. • Identify line and rotational symmetries (ACMMG114).
Year 5 Achievement standard
• They describe transformations of two-dimensional shapes and identify line and rotational symmetry. • Students compare and interpret different data sets. • Students use a grid reference system to locate landmarks. • They measure and construct different angles. • Students list outcomes of chance experiments with equally likely outcomes and assign probabilities between 0 and 1. • Students pose questions to gather data, and construct data displays appropriate for the data.
Year 6 Content, location and transformation
• Introduce the Cartesian coordinate system using all four quadrants (ACMMG143).
Year 7 Chance and data representation and interpretation
• Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168).
Year 8 Geometric reasoning
• Define congruence of plane shapes using transformations (ACMMG200). • (Elaboration) Establishing that two figures are congruent if one shape lies exactly on top of the other after one or more transformations (translation, reflection, rotation), and recognising that the matching sides and the matching angles are equal.

Figure 1. When examining the numbers and letters separately, it can be seen that the two licence plates are composed of two palindromes each.

One-to-one correspondence

Many children are familiar with secret codes and decoding devices. A popular movie from 1983, *A Christmas Story*, has an interesting scene where the main character receives a decoding device in the mail that can be used while listening to a radio program. On the radio show, a secret message would be broadcast and by matching the corresponding letters in the decoder, a secret message would be revealed. In mathematics class, we call this matching of one item in a set to an item in another set, a 'mapping'. For example, a simple mapping would let A→1, B→2, C→3 (this is read "A maps to 1, B maps to 2, C maps to 3), and so on through the alphabet. We can use this sort of thinking for a unique method of graphing licence plates, and as in the movie, we can find hidden mathematics as we create our graphs.

Graphing licence plates

Consider the licence plate shown in Figure 2. The licence plate is obviously not composed of any palindromes. In order to graphically represent licence plates, I have designed a piece of graph paper (see Figure 3, and Appendix A) where the horizontal axis records the six characters from the plate. The left side of the vertical axis is labelled with sequential numbers and the right side is labelled with sequential letters. The letters and numbers can be related to one another

sequentially; for this particular graph let L→0, M→1, N→2 all the way to X→12. This style of graphing becomes more interesting when particular licence plates with inherent mathematical properties are selected as demonstrated in the following sections.

Figure 2. A random licence plate.

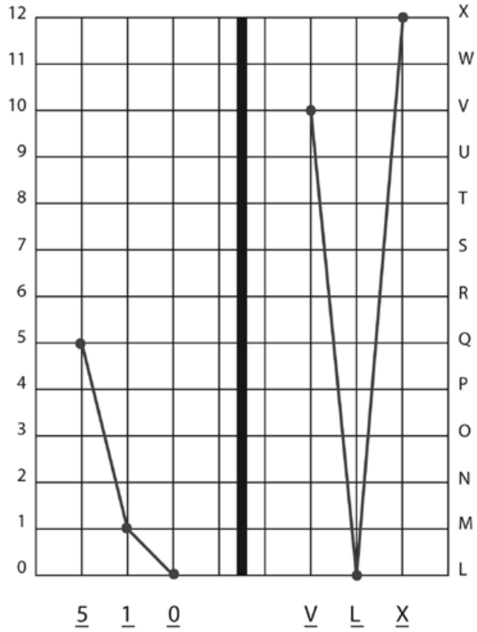

Figure 3. The graphical representation of licence plate 510 VLX.

Transformations

The mathematical study of transformations and automobiles has an interesting history. A classic booklet in teaching transformations, Kaleidoscopes, Hubcaps, & Mirrors (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), focused on automobiles in the *Connected Mathematics Project* textbook series and used activities with hubcaps to investigate rotational symmetry. With a focus on licence plates and the graphing technique just presented, investigations in translations, reflections, and rotations can easily be explored in the context of automobiles.

Translations

A translation is the transformation that simply moves one shape onto another without any flipping or rotating. Translations are considered the most basic of the transformations, and in the context of licence plates, having a palindrome licence plate provides a graph that allows a translation. One licence plate example for this is seen in Figure 4. This particular plate is mathematically interesting for two reasons. First, it is composed of a palindrome set of numbers and letters. Second, the letter H happens to be the 8th letter of the alphabet, and G happens to be the 7th letter in the alphabet. In order to graph this, we can establish the mapping that matches A→1, B→2, C→3, and so on until we arrive at G→7 and H→8. If you live in or near a state or province that uses the three letters and three numbers scheme for licence plates, you may have difficulty finding a palindromic plate of numbers and letters. However, finding one with this particular mapping of letters to numbers as they are ordered in the alphabet is extremely rare. For those inclined to see mathematics in such situations, I encourage you to take a photograph of the plates you find, as you may not see them very often.

Figure 4. A licence plate that reveals a translation between numbers and letters.

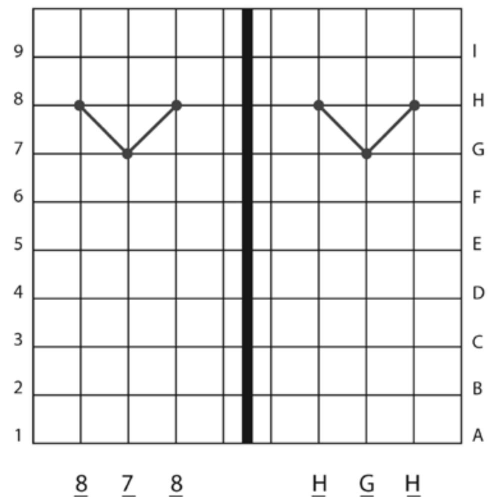

Figure 5. The graphical representation of licence plate 878 HGH.

As you can see from the graph in Figure 5, not only is a palindromic licence plate interesting to read in photograph form, but the graph of the licence plate is more visually appealing than the graph in Figure 3. Because of our choice in the mapping of letters and numbers, a simple horizontal translation of five units applied to the left graph reveals these two graphs to be identical. This graphical relationship cannot be seen when looking only at the licence plate.

Palindromic licence plates can be difficult to find, and mathematically this can be explained assuming random assignment of numbers and letters and calculating the probability (which is beyond the scope of this article). However, repeated digits are also easy to notice and they occur more often. Using this graphing technique allows us to look beyond translations to the other types of transformations taught in school mathematics.

Reflections

In the context of licence plates, the concept of reflection can be interpreted based on finding a line of reflection on one graph that will yield the other. Consider the licence plate in Figure 6, which I thought would be nice to have on a car because it would be easy to remember. Figure 7 displays how this will look using the mapping G→3, H→4, I→5, and so on.

Figure 6. A licence plate that reveals a glide-reflection between numbers and letters.

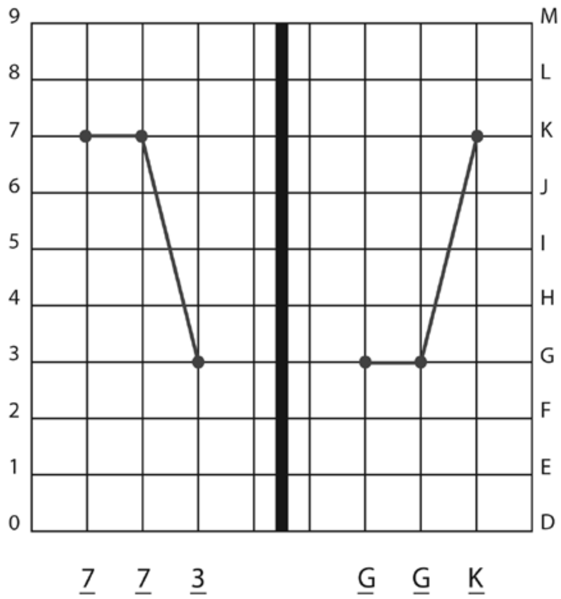

Figure 7. The graphical representation of licence plate 773 GGK.

A glide-reflection can translate the figure on the left side of the line to match the figure on the right. Starting with the left side figure, a horizontal glide of five units to the right, followed by a reflection along the horizontal line labelled '5' or 'I' reveals a match between these two figures.

Students may wonder how I knew that this particular licence plate would provide such an interesting graph. Part of this is chance; part of it is simple counting. Notice that this licence plate has repeating digits and repeating letters. These are easier to locate than plates without the repetition, and you will encounter these sorts of plates by chance fairly often. So one way to find plates with the glide-reflection property (or to create your own), is to find plates with repeating digits. Once you find one, you simply count the distance between the letters. In this case, from G to K is 4, which happens to be the distance from 3 to 7. Using mental mathematics, I know that this will be a nice graph. Further analysis, typically through graphing, is the easiest way to determine if the licence plate will require a glide-reflection or another transformation.

Rotations

Finally, rotations can be seen in licence plates such as 889 GHH, shown in Figure 8, using the mapping A→1, B→2, … , G→7, H→8, and so on. The graph on the left side of Figure 9 can be rotated 180 degrees to obtain the same orientation as the graph on the right. Once the figure is rotated, a translation of five units to the right and one unit down will allow the two figures to overlap. In class we say that the figure on the left can be rotated 180 degrees and shifted five spaces right and one unit down to match the graph on the right.

Figure 8. A licence plate that reveals a rotation between numbers and letters.

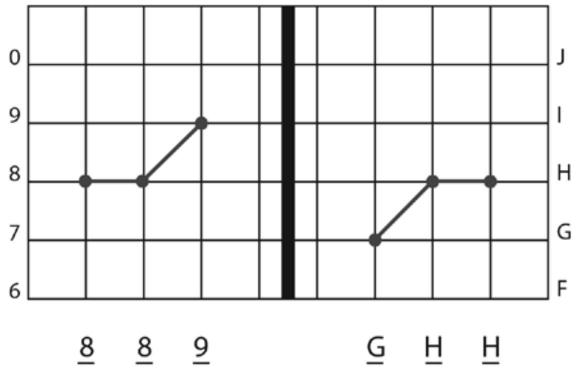

Figure 9. The graphical representation of licence plate 889 GHH.

Creating your own

One thing that the teacher needs to be clear on is the allocation of the mapping codes. Each licence plate will likely need a different letter and number mapping for each graphing. The default way is to map the earliest letter in the licence plate to the lowest number, and then in your graph, label the row before your graphing begins (see Figure 3). However, you are free to experiment with the mapping. Figure 9 shows how I mapped G→7, because G happens to be the 7th letter in the alphabet. Ultimately, the mapping is a free choice and should lead to some lively classroom discussions.

Once the techniques of graphing are learned, students enjoy creating their own. Remember, licence plates are interesting and many people with cars pay to have their plates personalised. Most regions have special group licence plates such as sports teams, non-profit organisations, or other groups that submit requests to have their own unique plates, as well as vanity plates, where a person can request a particular phrase to be displayed. Students will be eager to engage in the following classroom activity as shown in Figure 10. If students have difficulty getting started, students can consider birth dates, initials, favourite numbers, or use the licence plate provided in Figure 11.

Create your own licence plate
What style is your plate? 3 numbers |3 letters, specialty, vanity, or your home state/province.
What is mathematical about your plate? Use of palindromes, transformations, something personal.
We will share in about 15 minutes.

Figure 10. Instructions for a classroom activity.

The nature of this activity lends itself to a gallery walk, an excellent activity described by Van de Walle, Karp, and Bay-Williams (2013) to allow student work to be displayed. It is important that students' work be displayed, especially in mathematics class. If we desire our students to develop their productive disposition toward mathematics, activities such as this provide a fantastic opportunity.

Figure 11. A South Australia licence plate with palindromes.

Conclusion

We live in a mathematically rich environment, one in which the Pythagoreans would feel at home in. So many things in our society are labeled and numbered, and cars and other vehicles prominently display license plates all around the world. Previously, I have shown other ideas for working with palindromes in novel ways, such as studying patterns of products of palindromes, investigating the occurrences of prime palindromes, and writing palindrome words horizontally or vertically in particular fonts (Nivens, 2013).

In an era of evolving standards, some teachers feel that creativity is squelched. Others view the standards as an opportunity to engage students in content that simply needs to be explicit in exciting and engaging contexts. The students and teachers that I have shared this with have been excited to study the concept of transformations within the context of licence plates with palindrome sequences. Previous authors Hildebrandt, Biglan, and Budd (2013) have found students to be very engaged in mathematical activities within the context of licence plates. My hope is that your students will enjoy working with transformations.

Acknowledgements

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Appendix A

Blank template for use in the classroom.